EPJ manuscript No.
(will be inserted by the editor)

Cosmological Constraints on Unparticle Dark Matter

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Received: March 20th, 2008

Abstract. In unparticle dark matter (unmatter) models the equation of state of the unmatter is given by $p = \rho/(2d_U + 1)$, where d_U is the scaling factor. Unmatter with such equations of state would have a significant impact on the expansion history of the universe. Using type Ia supernovae (SNIa), the baryon acoustic oscillation (BAO) measurements and the shift parameter of the cosmic microwave background (CMB) to place constraints on such unmatter models we find that if only the SNIa data is used the constraints are weak. However, with the BAO and CMB shift parameter data added strong constraints can be obtained. For the ΛUDM model, in which unmatter is the sole dark matter, we find that $d_U > 60$ at 95% C.L. For comparison, in most unparticle physics models it is assumed $d_U < 2$. For the ΛCUDM model, in which unmatter co-exists with cold dark matter, we found that the unmatter can at most make up a few percent of the total cosmic density if $d_U < 10$, thus it can not be the major component of dark matter.

1 Introduction

It has recently been proposed that a hidden scale-invariant sector of matter may exist [1], and is named "unparticle" for its unusual behavior. In this scenario, there is a scale invariant sector with a non-trivial infrared fixed point, called the Banks-Zaks (BZ) field [2]. The BZ field interacts with the standard model (SM) fields via exchange of particles of mass M_U:

$$\mathcal{L}_{BZ} = \frac{O_{BZ} O_{SM}}{M_U^k},$$

where O_{BZ} is the BZ operators with mass dimension d_{BZ} and O_{SM} is the SM operators with mass dimension d_{SM}. Dimensional transmutation produce a scale Λ_U, below which

$$\mathcal{L}_{BZ} \to \mathcal{L}_U = C_U \frac{\Lambda_U^{d_{BZ}-d_U}}{M_U^k} O_{SM} O_U,$$

where C_U is the coefficient function and O_U is the unparticle operators with mass dimension d_U. Phenomenological constraints on M_U, Λ_U, C_U, d_U have been derived from a number of particle physics [3,4,5,6,7,8,9,10,11,12,13,14,15,16,17] and astrophysics [18,19,20,21,22,23,24] analysis.

Given that the unparticles interacts weakly with standard model particles, it is natural to consider unparticle matter, or *unmatter*, as a candidate of dark matter, especially if the unparticle could be stabilized by a discrete symmetry [25]. Thanks to the unusual kinematics of unparticles, the behavior of unmatter is distinctly different from the usual cold dark matter.

The unparticles do not have a fixed mass, and the density of states of unparticle is given by

$$\frac{d^4p}{(2\pi)^4} 2p^0 \theta(p^0)\theta(p^2)(p^2)^{d_U-2}. \tag{1}$$

Typically $1 < d_U < 2$ are considered [1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24]. For a thermal distribution of unparticles, the density and pressure is given by [26]

$$p_U = g_s T^4 \left(\frac{T}{\Lambda_U}\right)^{2d_U-1} \frac{\mathcal{C}(d_U)}{4\pi^2} \tag{2}$$

$$\rho_U = (2d_U+1) g_s T^4 \left(\frac{T}{\Lambda_U}\right)^{2d_U-1} \frac{\mathcal{C}(d_U)}{4\pi^2} \tag{3}$$

where $\mathcal{C}(d_U) = B(3/2, d_U)\Gamma(2d_U+2)\zeta(2d_U+2)$, and B, Γ, ζ are the Beta, Gamma and Zeta functions. The equation of state for the unmatter is therefore

$$w_U = 1/(2d_U+1) \tag{4}$$

Thus, as the universe expands, the energy density of unmatter evolves as $\rho_U(z) = \rho_{U0}(1+z)^{3(1+w_U)}$. If $d_U = 1$, this is the same as radiation, and for $d_U \to \infty$ its behavior would be similar to cold dark matter. In the intermediate case, its evolution would differ from both radiation and cold dark matter. We can then use cosmological observations to constrain the value of d_U.

We shall consider two models. In both cases we assume the universe is flat with a cosmological constant. In the first case, denoted by ΛUDM, the unmatter serves

as the sole dark matter, and the set of cosmological parameters θ is $\{\Omega_{b0},\ \Omega_{U0},\ d_U,\ h_0\ \}$. In the second case, denoted by ΛCUDM, we consider the more general case where the unmatter is not the only source of dark matter. We then constrain the amount of unmatter if it does exist. The cosmological parameters set θ in this case is $\{\Omega_{m0},\ \Omega_{U0},\ d_U,\ h_0\}$. We then use a Markov Monte Carlo Chain method to make the global fitting and constraints. For details of our MCMC code we refer the readers to Ref. [27].

We consider three observational constraints. The first one is luminosity distance moduli to type Ia supernovae (SNIa). The second is the baryon acoustic oscillation (BAO) feature in large scale structure as measured by the Sloan Digital Sky Survey (SDSS) and the Two Degree Field Galaxy Redshift Survey (2dFGRS). The last one is the so called shift parameter[28,29], which is essentially a measure of the distance to the last scattering surface of the cosmic microwave background (CMB), as measured by the WMAP three year observation [30]. All of these provide constraints on the global expansion history of the universe.

2 Methods

The cosmic expansion rate $H(z)$ is given by

$$H^2(z) = H_0^2\ \Omega(\mathbf{z};\theta), \tag{5}$$

where for ΛUDM:

$$\begin{aligned}\Omega(\mathbf{z};\theta) &= \Omega_{b0}(1+z)^3 + \Omega_{\Lambda 0} + \Omega_{r0}(1+z)^4 \\ &\quad + \Omega_{U0}(1+z)^{3(1+w_U)}\end{aligned} \tag{6}$$

and for ΛCUDM:

$$\begin{aligned}\Omega(\mathbf{z};\theta) &= \Omega_{m_0}(1+z)^3 + \Omega_{\Lambda 0} + \Omega_{r0}(1+z)^4 \\ &\quad + \Omega_{U0}(1+z)^{3(1+w_U)};\end{aligned} \tag{7}$$

with $\Omega_{\Lambda 0} = 1-\Omega_{m0}-\Omega_{U0}-\Omega_{r0}$. Here, $\Omega_{m0}, \Omega_{r0}, \Omega_{\Lambda 0}, \Omega_{U0}$ are the relative abundance of matter, radiation, the cosmological constant, and unmatter respectively. Of course, for the ΛUDM model, $\Omega_{m0} = \Omega_{b0}$.

Supernova constraint: the luminosity distance to a supernova is given by

$$d_L(z;\theta) = (1+z)\int_0^z \frac{c dz'}{H(z')}. \tag{8}$$

and the distance moduli is

$$\mu_{th}(z) = 5\log_{10} d_L(z) + 25, \tag{9}$$

The χ^2 for the SNIa data is

$$\chi^2_{\rm SN}(\theta) = \sum_{i=1}^{N} \frac{(\mu_{obs}(z_i) - \mu_{th}(z_i))^2}{\sigma_i^2}, \tag{10}$$

where $\mu_{obs}(z_i)$ and σ_i are the observed value and the corresponding error for each supernova. We use a data set of 182 high-quality SNIa [27] selected from the Gold06 [32], SNLS [33] and ESSENCE [34] samples.

CMB constraint: the CMB shift parameter R [28] denotes the positions of the acoustic peaks in the angular power spectrum of CMB, and takes the form as

$$\mathrm{R} = \sqrt{\Omega_{m0}}\int_0^{z_{\rm CMB}} \frac{dz'}{H(z')/H_0} \tag{11}$$

The WMAP3 data gives $\mathrm{R} = 1.70 \pm 0.03$ [29], thus we have

$$\chi^2_{\rm R} = \left(\frac{\mathrm{R} - 1.70}{0.03}\right)^2. \tag{12}$$

BAO constraint: we use the quantity $\rm r_s/D_v$ which is constrained by the BAO signature in SDSS (at $z = 0.35$) and 2dFGRS (at $z = 0.2$) data [35,36]: $\rm r_s/D_v(0.2) = 0.1980 \pm 0.0058$ and $\rm r_s/D_v(0.35) = 0.1094 \pm 0.0033$, with a correlation coefficient of 0.39. Here $\rm r_s$ is the comoving sound horizon size at the epoch of decoupling, and $\rm D_v$ is the effective distance defined in [31]. we do not use the parameter A which is extracted from the BAO measurements of the SDSS, as its definition applies to the ΛCDM model specifically, and may not be applicable in the presence of unmatter models [37].

For the combined analysis,

$$\chi^2 = \chi^2_{\rm SN} + \chi^2_{\rm R} + \chi^2_{\rm BAO}. \tag{13}$$

We employ the Markov Chain Monte Carlo (MCMC) technique to calculate the posterior probability distributions function of the parameters. The Metropolis-Hastings algorithm with uniform priors is used to generate the sample, and the priors are taken as the following: $\Omega_{b0} \in (0, 0.1)$, $\Omega_{m0} \in (0, 1)$, $\Omega_{U0} \in (0, 1)$, $d_U \in (0, 10^5)$ and $h_0 \in (0.4, 0.9)$. The energy density of all components are assumed to be positive, $\Omega_{U0} \in (0, 1 - \Omega_{b0}/\Omega_{m0} - \Omega_{r0})$ is set so that $\Omega_{\Lambda 0} = 1 - \Omega_{U0} - \Omega_{b0}/\Omega_{m0} - \Omega_{r0} \geq 0$. For each of the two models (ΛUDM and ΛCUDM) we generate six chains, and about ten thousands points are sampled in each chain. After thinning the chains, we merge them into one chain which consists of about 10000 points.

3 Results

First we consider the constraints derived purely from the SNIa data. For constraining the unmatter model, this is the most reliable one, as it is based only on the global expansion history, which can be calculated exactly for the given parameter set.

In Fig. 1 we plot the constraint on Ω_U and d_U in the ΛUDM model after marginalizing the other parameters. We found that practically all values of d_U are allowed. At large values of d_U, the central value of Ω_U is between 0.15 and 0.25. This is what we would have expected, since for large value of d_U the behavior of the unparticle gas is very similar to that of the cold dark matter, and for ΛCDM the best fit is centered in the same region. At smaller values of d_U, the contours curved to smaller Ω_U.

In Fig. 2 we plot the constraint on Ω_U and d_U in the ΛCUDM model. Again, practically all values of d_U are allowed. For large d_U where the unmatter asymptotes to cold dark matter, the best fit is located at $\Omega_U \sim 0.22$, as the UDM become dominant and the CDM has a very small abundance. However, for high values of d_U, all values of Ω_U are allowed, in this part of the parameter space the UDM plays a minor role and the CDM is dominant.

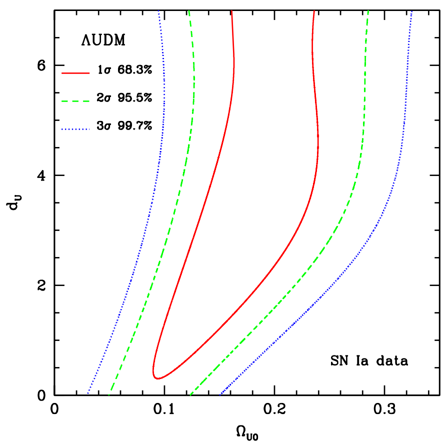

Fig. 1. The $1\sigma, 2\sigma, 3\sigma$ contours of the Ω_U and d_U in the ΛUDM model derived from SNIa observations.

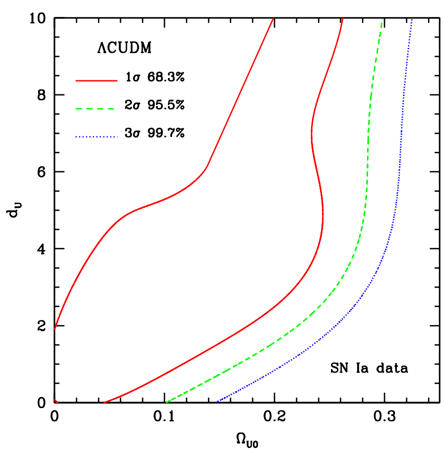

Fig. 2. The $1\sigma, 2\sigma, 3\sigma$ contours of the Ω_U and d_U in the ΛCUDM model derived from SN Ia observations.

Our constraint is improved significantly by using the BAO and CMB shift parameter in addition to the SNIa data. The $\mathrm{r_s/D_v}$ proposed by Percival et al. [35] is a ratio of two "standard ruler", which is suitable and reliable to constrain our unmatter model and could efficiently break the degeneracy of the parameters. The shift parameter does not include all information in the CMB angular power spectrum, but it is relatively easy to compute for unconventional models. The large distance to the last scattering surface of the CMB provides a long level arm for constraining the global expansion rate. We plot the $\Omega_U - d_U$ contours for the ΛUDM model in Fig. 3, and the same contours for the ΛCUDM in Fig. 4. As can be seen from the figures, the distribution is drastically different from the SNIa only constraints.

For the ΛUDM model, the lowest edge of the 2σ (95.5% C.L.) contour is about 60. Thus, small values of d_U, which is interesting from a physics perspective, is excluded at high confidence levels. At large d_U the unmatter is similar to cold dark matter, so again it is not surprising that the center value of Ω_U is 0.23-0.29.

For the ΛCUDM model the constraint is also much more stringent. At 2σ level, only 2 or 3 percent of the total cosmic density could be in unmatter. For the scaling dimension d_U, the minimal allowed value is 1.2 at 95.5%C.L.. Furthermore, for the small d_U values, the constraint on the unmatter density is stronger. For $d_U < 2$, the limit on unmatter density is $\Omega_U < 0.01$ at 2σ level. The constraint on abundance Ω_U loosened at greater values of d_U, where the unmatter become indistinguishable from cold dark matter.

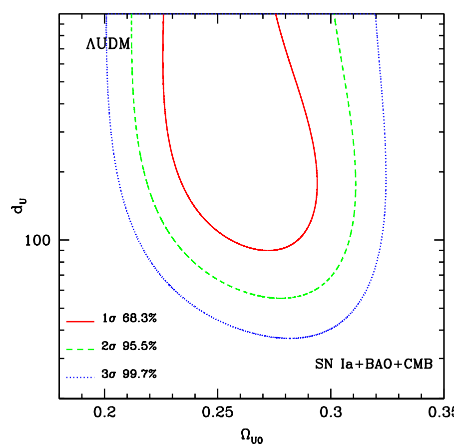

Fig. 3. The $1\sigma, 2\sigma, 3\sigma$ contours of the Ω_U and d_U in the ΛUDM model derived from SNIa, BAO and CMB shift parameter observations.

4 Conclusion

In this work, the scaling dimension d_U and the abundance Ω_U of the unparticle dark matter are constrained by using the SNIa luminosity distance moduli, BAO features in large scale structure as measured by SDSS and 2dFGRS, and the CMB shift parameter which depends on the cosmic expansion history. We used the MCMC technique to simulate the posterior probability distributions of the parameters. Two models are considered, viz. ΛUDM (where unparticles are the only dark matter) and ΛCUDM (where unparticles co-exists with other cold dark matter).

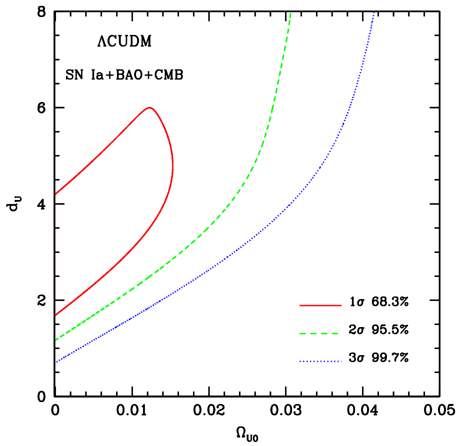

Fig. 4. The $1\sigma, 2\sigma, 3\sigma$ contours of the Ω_U and d_U in the ΛCUDM model derived from SNIa, BAO and CMB shift parameter observations.

Using only the supernova data, we find that the constraints on d_U and Ω_U are pretty weak. However, with the addition of the BAO and CMB shift parameter data, the degeneracy is broken, and strong constraint could be put on d_U and Ω_U. For the ΛUDM model, $d_U > 60$ at 2σ level. For the ΛCUDM model, if $d_U < 2$, then $\Omega_U < 0.01$ at 95.5% C.L.; and Ω_U is no greater than a few percent if $d_U < 10$. These limits severely constrained models of unparticle dark matter and stable relic unparticle matter.

A major assumption adopted in this work is the equation of state w_U for the unmatter: $w_U = 1/(2d_U+1)$. This expression was derived by S. L. Chen et al in Ref. [26] for a thermal distribution of unparticle matter. This result is a little surprising, since an ideal gas of massless particles which also possesses scale invariance has an equation of state $w_U = 1/3$. However, if one accepts the usual density of states for the unparticles (Eq. 1), which was used for other calculations about unparticles, e.g. production of unparticle in particle collisions, then this equation of state can be derived using the standard method of statistical mechanics, as has been shown in Ref. [26]. It is also possible that the unparticle matter has a non-thermal distribution, then its equation of state would be different, and in that case it is not constrained by our result. However, if the unparticle matter was produced during the hot Big Bang in the standard decoupling scenario, its very nature must be considered a *thermal* distribution of unparticle matter.

We thank professors Miao Li, Xiaogang He and Yi Ling for discussions. Our MCMC chain computation was performed on the Supercomputing Center of the Chinese Academy of Sciences and the Shanghai Supercomputing Center. This work is supported by the National Science Foundation of China under the Distinguished Young Scholar Grant 10525314, the Key Project Grant 10533010, by the Chinese Academy of Sciences under the grant KJCX3-SYW-N2, and by the Ministry of Science and Technology national basic science Program (Project 973) under grant No. 2007CB815401. X.C. also acknowledges the hospitality of the Moore Center of Theoretical Cosmology and Physics at Caltech and Kavli Institute for Theoretical Physics in China, where part of this research was performed.

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